SUPPRESSION OF SPONTANEOUS FLUCTUATIONS IN AMPLIFIERS AND RECEIVERS FOR ELECTRICAL COMMUNICATION AND FOR MEASURING DEVICES

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Zusammenfassung

In der Einleitung wird auf die grundlegende Bedeutung des übertragenen Frequenzbandes bei Empfängern und Verstärkern hingewiesen. Abschn. II enthält die Berechnung des Einflusses einer beliebigen Rückkopplung auf das Verhältnis der Signal- zur Schwankungsspannung am Ausgang eines linearen Vierpols. Es wird gezeigt, dass dieses Verhältnis sich durch Rückkopplung nicht ändert. In Abschn. III wird die Eingangsschaltung eines Verstärkervierpols im Hinblick auf das genannte Verhältnis betrachtet. Man kann dieses für eine bestimmte Frequenz steigern, indem der Eingangswiderstand des Vierpols gesteigert wird, wie aus Abschn. IV hervorgeht. Durch nachträgliche Anwendung einer geeigneten Rückkopplung kann die Frequenzkennlinie des Vierpols wieder auf den ursprünglichen Wert gebracht werden, während das hohe Verhältnis der Signal- zur Schwankungsspannung erhalten bleibt (Abschn. V). Abschn. VI enthält die Anwendung dieses Verfahrens auf Messeinrichtungen für winzige Ströme und Ladungen.

I. *Introduction.* It has been shown, that in several cases methods may be applied, which aim at a nearly complete suppression of the noise compared with the signal strength in the output leads of amplifiers or receivers [2] [9] [10] [11] [14] [15]. Therefore it may be useful to give a more *general* treatment of these methods. We also take the opportunity to apply these methods to measuring devices.

It is essential for our arguments, that the fundamental rôle of the frequency band to be transmitted is fully understood. An unvariable signal of a single frequency can indeed convey no message at all. Any sort of communication implies an alteration of the transmitted signal and hence a change of frequency. This change may be slow, if the

speed of signalling is low. It may be fast, if the speed of signalling is high. This speed of signalling or message volume transmitted within a certain lapse of time is directly proportional to the frequency band used in the transmission. The amplifier or receiver used in the transmission has to be adapted to the required frequency band: all frequencies within this band B should be equally amplified or received.

II. *Signal to noise ratio of a linear four-terminal network with linear feedback.* In this section the influence of feed back on the signal-to-noise ratio of an amplifier will be analysed, resulting in a general theorem, which will be applied in the sections IV and V.

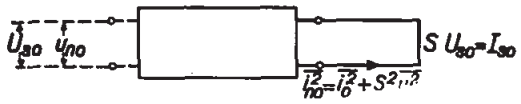

Fig. 1. Four-terminal amplifying network with input signal voltage U_s, input fluctuation (noise) voltage u_n, output signal current $I_s = SU_s$ and output fluctuation current $i_o + Su_n$.

We consider (fig. 1) a general linear four-terminal network without internal feedback, which may include amplifying valves and impedances of any kind. Without loss of generality the output terminals may be assumed to be short-circuited. In the output lead flows a signal current $I_{so} = SU_{so}$, where U_{so} is the signal voltage between the input terminals and S is an admittance, proper to the network. If only one amplifying valve is used in the network, S is proportional to the mutual conductance of the valve. The output fluctuating current in a small frequency band Δf, small compared with the bandwidth B, is i_{no}. This current is composed of two parts, one part is i_o, which is also present if the input is short-circuited, and a second part Su_{no}, due to the fluctuation-(noise-)voltage u_{no} between the input terminals. The two parts of i_{no} are uncorrelated and hence add according to $\overline{i_{no}^2} = \overline{i_o^2} + S^2\,\overline{u_{no}^2}$.

We now take into account an external negative or positive feedback [1] [12] [13]. Hereby the input signal voltage which was equal to U_{so} prior to the application of the feedback is altered to U_s. The action of the feedback may be described by the relation:

$$U_s = U_{so} + \beta S U_s \tag{1}$$

or

$$U_s = \frac{U_{so}}{1 - \beta S}, \tag{1a}$$

where β is a quantity having the dimensions of an impedance, denoting the amount of feedback.

By applying feedback, the input impedance of the four-terminal network is altered from a value Z_{io} prior to the feedback to a value Z_i after the feedback. If a current I is flowing into the input of the network, the voltage U_{so} is given by $IZ_{io} = U_{so}$ without feedback. *With* feedback we have:

$$IZ_i = U_s = \frac{U_{so}}{1 - \beta S}$$

hence:

$$Z_i = \frac{Z_{io}}{1 - \beta S} . \tag{2}$$

The feedback also causes an alteration of the input noise voltage from the value u_{no} before to u_n after the feedback and hence:

$$u_n = \frac{u_{no}}{1 - \beta S} . \tag{3}$$

Furthermore, an additional noise voltage u_v arises at the input, due to the feedback of the output fluctuation current i_o:

$$u_v = (i_o + S u_v)\, \beta$$

or

$$u_v = \frac{i_o \beta}{1 - \beta S} . \tag{4}$$

The signal to noise ratio prior to the feedback is:

$$\frac{I_{so}^2}{\overline{i_{no}^2}} = \frac{S^2 U_{so}^2}{\overline{i_o^2} + S^2 \overline{u_{no}^2}} . \tag{5}$$

With the feedback in effect, we have an output signal current by (1*a*):

$$I_s = S U_s = \frac{S U_{so}}{1 - \beta S}$$

and an output noise current derivable from (3) and (4)

$$\overline{i_n^2} = S^2 \overline{u_n^2} + \overline{(S u_v + i_o)^2} = \frac{\overline{S^2 u_{no}^2} + \overline{i_o^2}}{|\, 1 - \beta S \,|^2} .$$

Hence we have an output signal to noise ratio with feedback:

$$\frac{I_s^2}{\overline{i_n^2}} = \frac{S^2 U_{so}^2}{\overline{i_o^2} + S^2 \overline{u_{no}^2}} . \tag{6}$$

Comparing (5) and (6) we find the important theorem that *the signal to noise ratio at the output of any linear four-terminal network is not altered by the application of any linear positive of negative feedback* *).

III. *Equivalent circuit for the input of the amplifier or receiver.* Any h.f. or l.f. amplifier or receiver amplifying a frequency band of width B contains at least one amplifying valve. In the case of low frequency amplification the input circuit of this valve mainly consists of a resistance R, connecting the grid to ground for direct current, with the wiring capacity C_1 and the valve capacity C_e parallel to R. In a h.f. amplifier the input circuit will be assumed to be tuned to a frequency corresponding with the center of the frequency band used in transmission (fig. 2*a*). Obviously the circuit represented in

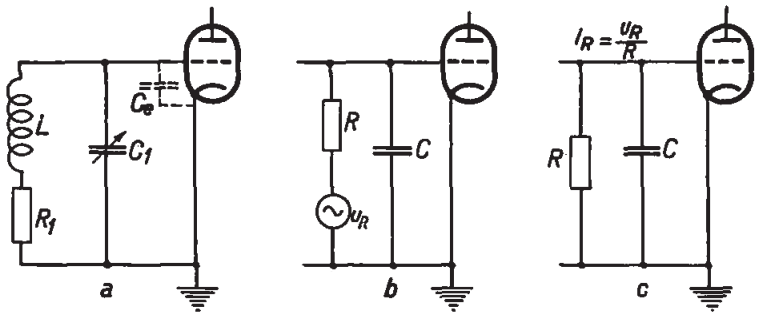

Fig. 2*a*. Circuit containing a selfinduction L in series with a resistance R_1 and a tuning capacity C_1 connected to the grid of a valve with input capacity C_e. The circuit is tuned at an angular frequency ω_0 according to

$$\omega_0^2 L(C_1 + C_e) = 1.$$

Fig. 2*b*. Equivalent circuit of fig. 2*a* for a frequency $2\Delta\omega = 2|\omega_0 - \omega|$ ($\Delta\omega \ll \omega_0$) . $C = (C_1 + C_e)$, R is the impedance of the tuned circuit ($\Delta\omega = 0$). u_R is a fluctuating e.m.f. in series with R due to the Brownian motion of electricity in R.

Fig. 2*c*. Circuit equivalent to that of fig. 2*b*, u_R being replaced by a fluctuating current flowing into the grid circuit.

fig. 2 by L, R_1, C_1, C_e (where L denotes the selfinduction of the circuit, R_1 is connected with the ohmic losses of the coil and C_1 denotes the tuning capacity) is very nearly aequivalent to a single ohmic resistance R for the angular frequency $\omega = \omega_0$, where $\omega_0^2 L(C_1 + C_e)$

*) As Prof. van der Pol informed us, this theorem was suggested by him in 1934 and confirmed experimentally by Ir. Ziegler in the same year. Compare also several publications [14] [6] [13] [9] [10] [11].

$= 1$. This resistance R amounts to $R = L/R_1(C_1 + C_e)$. If $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega$ may be positive or negative and $\Delta\omega/\omega_0 \ll 1$, it may be shown, that the impedance of the tuned circuit for the angular frequency ω is nearly equal to the impedance of R, C_1 and C_e, connected parallel to each other, for the much lower angular frequency $2\Delta\omega = 2|\omega - \omega_0|$. Therefore it is sufficient to restrict our calculations to l.f. amplifiers with an input circuit containing a resistance R and a capacity $C = (C_1 + C_e)$ (fig. 2*b*).

We now turn to the fluctuating voltage connected with this circuit. The resistance R, showing minute voltage fluctuations at its terminals, may be represented by a fluctuating e.m.f. u_R in series with R (fig. 2*b*) or by an external fluctuating current $i_R = u_R/R$ flowing into the input circuit (fig. 2*c*). It is well known, that the mean square of u_R and i_R for a frequency interval Δf are given [7]) by the relations:

$$\overline{u_R^2} = 4kTR\Delta f \tag{7a}$$

and

$$\overline{i_R^2} = \frac{4kT}{R}\Delta f. \tag{7b}$$

This also holds approximately for the h.f. circuit of fig. 2*a*, if R is the impedance of the tuned circuit for the frequency ω_0.

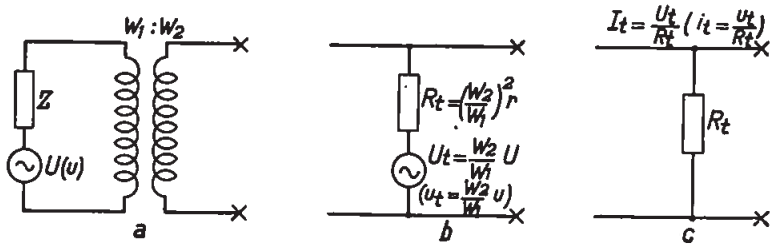

Fig. 3*a*. Voltage source U with fluctuating voltage u and internal resistance r (erroneously called Z in the fig.), connected to an ideal transformer of turns ratio W_2/W_1.

Fig. 3*b*. Equivalent circuit of fig. 3*a*, r, U and u being replaced by their transformed values R_t, U_t and u_t.

Fig. 3*c*. Circuit equivalent to that of fig. 3*b*, U_t and u_t being replaced by currents I_t and i_t flowing into the circuit.

The signal voltage source, connected to the input of the amplifier by an ideal transformer of turn ratio (W_2/W_1), may be described as an e.m.f. U in series with a resistance r (fig. 3*a*). For this voltage source with transformer, equivalent circuits are given in fig. 3*b* and 3*c*. In fig. 3*b* the circuit of fig. 3*a* is replaced by an e.m.f. $U_t = (W_2/W_1)\,U$ in series with a resistance $R_t = (W_2/W_1)^2\,r$; in fig. 3*c*

the circuit of fig. 3*a* is replaced by an external current:

$$I_t = \frac{U_t}{R_t} = \left(\frac{W_1}{W_2}\right)\frac{U}{r} \tag{8}$$

flowing into the circuit. The fluctuating voltage connected with the signal is denoted by u in fig. 3*a*, by u_t in fig. 3*b* and the fluctuating current by i_t in fig. 3*c*.

The spontaneous fluctuations of the rest of the amplifier or receiver may be represented by an equivalent fluctuation voltage u_e in the grid circuit. After inserting this voltage the amplifier may further be considered ideal, i.e. amplifying without any more fluctuations. This voltage u_e in the small frequency interval Δf is given by:

$$u_e = \frac{i_o}{S} \text{ (i_o and S being defined in section II)} \tag{9}$$

or by an equivalent resistance R_e, which is called „noise resistance" given by:

$$\overline{u_e^2} = 4kTR_e\Delta f, \tag{9a}$$

T being room temperature, R_e in most cases depending on the first amplifying tube only.

Under the foregoing assumptions our input circuit may now be completely represented as regards impedances and voltages by fig. 4. In fig. 4 the influence of feedback is described by the impedance Z_f connected parallel to the impedance Z_{io} of section II. We have:

$$\frac{1}{Z_{io}} = \frac{1}{R_t} + \frac{1}{R} + j\omega C, \tag{10}$$

and hence, according to (2):

$$\frac{1}{Z_f} = -\frac{\beta S}{Z_{io}} = \frac{1}{R_f} + j\omega C_f; \tag{10a}$$

R_f and C_f may be positive or negative.

The relation between the impedances and the bandwidth B is given by:

$$2\pi B = \frac{\left(\frac{1}{R_t} + \frac{1}{R} + \frac{1}{R_f}\right)}{(C + C_f)}. \tag{11}$$

In fig. 4 the signal to noise ratio of the receiver or amplifier is given by $U_s^2/\overline{u_s^2}$, where u_s stands for $(u_n + u_v + u_e)$.

In the equivalent circuit represented in fig. 4 the signal to noise ratio at the input is equal to the same ratio at the output, and this therefore holds for any kind of feedback. *We have thus arrived at a simple proof of the theorem of section II (compare footnote there).*

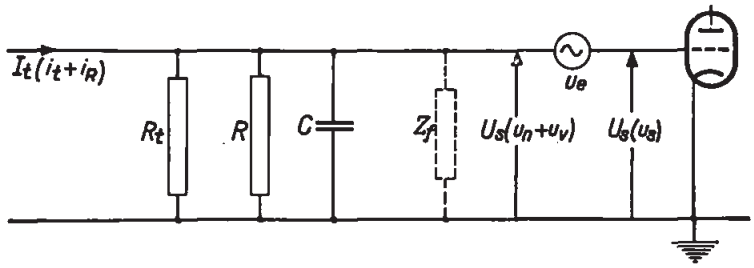

Fig. 4. Circuit equivalent to that of the input of an amplifier with feedback. R_t, R, C, I_t, i_t and i_R have the same meaning as in fig. 2 and 3. u_e is the equivalent noise voltage of the valve, Z_f denotes the impedance parallel to the input circuit due to feedback. U_s is the signal voltage on the grid. u_n is the fluctuating voltage at the input due to i_R and i_t. u_v is the fluctuating voltage in the input circuit, caused by the feedback and due to the shot effect of the valve. $u_s = (u_e + u_n + u_v)$.

IV. *Optimal signal to noise ratio without feedback.* We now aim at an optimal signal to noise ratio without feedback. This means $Z_f = \infty$, $u_v = 0$, $U_s = U_{so}$ and $u_s = u_{so} = u_{no} + u_e$. As the fluctuations of the voltage source do not materially affect our conclusions, they will be neglected here, therefore $i_t = 0$ and $u_{no} = i_R Z_{io}$. We will also neglect the capacity C, confining us to the lowest frequencies of the band. Afterwards the influence of these neglections will be discussed.

The input signal voltage is given by:

$$U_{so} = I_t Z_{io} = \frac{W_1}{W_2}\frac{U}{r}\left[\left(\frac{W_1}{W_2}\right)^2\frac{1}{r}+\frac{1}{R}\right]^{-1}, \tag{12}$$

the signal to noise ratio by:

$$\frac{U_{so}^2}{\overline{u_{so}^2}} = \frac{I_t^2 Z_{io}^2}{\overline{i_R^2} Z_{io}^2 + \overline{u_e^2}} = \frac{\left(\frac{W_1}{W_2}\right)^2\frac{U^2}{r^2}\left[\left(\frac{W_1}{W_2}\right)^2\frac{1}{r}+\frac{1}{R}\right]^{-2}}{\frac{\overline{u_R^2}}{R^2}\left[\left(\frac{W_1}{W_2}\right)^2\frac{1}{r}+\frac{1}{R}\right]^{-2}+\overline{u_e^2}}. \tag{13}$$

The input voltage is optimal for $R = r(W_2/W_1)^2$, the signal to

noise ratio, however, is an *optimum* for:

$$\left(\frac{W_2}{W_1}\right)^2 = \frac{R}{r}\left(1 + \frac{\overline{u_R^2}}{\overline{u_e^2}}\right)^{-\frac{1}{2}} \doteq \frac{R}{r}(1+\alpha)^{-\frac{1}{2}}\left(\text{if } \alpha = \frac{\overline{u_R^2}}{\overline{u_e^2}} = \frac{R}{R_e}\right), \quad (14)$$

its value then being given by:

$$\left(\frac{U_{so}^2}{\overline{u_{so}^2}}\right)_{opt} = \frac{U^2}{\overline{u_e^2}}\,\frac{R}{r}\,\frac{\sqrt{1+\alpha}}{\alpha + (1+\sqrt{1+\alpha})^2}. \quad (15)$$

The signal to noise ratio according to eq. (15) *is proportional to* $\sqrt{R}$ *for large values of* R *and therefore increases with increasing* R, so that noise may be almost entirely suppressed. A similar result was obtained some years ago by Williams [15]).

If the signal source may be considered as a current source of very high internal impedance, connected directly to the input circuit of the amplifier, $W_1/W_2 = 1$ and $U/r = I$, where I denotes the signal current. Then eq. (13) yields:

$$\frac{U_{so}^2}{\overline{u_{so}^2}} = \frac{I^2R^2}{\overline{u_R^2} + \overline{u_e^2}} = \frac{I^2R^2}{4kT\,(R+R_e)\Delta f}, \quad (13a)$$

which is directly proportional to R for $R \gg R_e$.

We now turn to the different neglections being made in our calculations. First we have neglected valve losses. If these losses are introduced, it means that the total impedance of the input circuit cannot be indefinitely increased, but will attain an upper value, determined by the valve losses. Therefore valve losses should be as low as possible.

If the fluctuations due to the voltage source (antenna, photocell etc.) cannot be neglected, this means that the highest value of the signal to noise ratio is $U^2/\overline{u^2}$. This also means that an increase of R above a certain upper value will not have much effect.

For the highest frequencies of the band, the circuit capacity C cannot be neglected. A calculation similar to the above one yields, that eqs. (14) and (15) are also valid in this case, if α is replaced by $(\alpha + \omega^2C^2R^2)$. From these results we may conclude, that the value of $(W_2/W_1)^2$ leading to an optimal signal to noise ratio, is smaller for the higher frequencies than for the lower ones. Hence it will be useful in many cases to choose $(W_2/W_1)^2$ somewhat smaller than the value of eq. (14), in order to improve the signal to noise ratio at the high frequency end of the frequency band.

If $\omega^2 C^2 R^2 \gg (1 + \alpha)$, the new eq. (15) transforms into:

$$\left(\frac{U_{so}^2}{u_{so}^2}\right) = \tfrac{1}{2} \frac{U^2}{u_e^2} \frac{1}{\omega C r}. \qquad (15a)$$

According to eq. (15*a*), the influence of the capacity C may be greatly diminished by making the value of C as low as possible whereas a low value of the equivalent „noise resistance" R_e of the first valve is also of interest.

V. *Applications of feedback. Examples.* In eq. (11) a relation is given between the parallel resistance, parallel capacity and the bandwidth B of the circuit, so that one of these quantities may be calculated, if the others are given. In practice, however, the bandwidth as well as the parallel capacity are given, whereas the parallel resistance ought to be chosen as high as possible at the cost of a decrease of U_{so} for the highest frequencies of the band B, in order to obtain a high signal to noise ratio. Thereupon feedback may be applied, restoring the bandwidth to the desired value B, without disturbing the obtained high signal to noise ratio.

From eq. (11) two distinct methods can be derived. First, the feedback may be chosen such that $Z_f = R_f$ is purely ohmic. This means, that *negative* feedback is applied. On the other hand the feedback may result in an impedance Z_f behaving like a negative capacity. Thereby $(C + C_f)$ may be so much decreased that eq. (11) is satisfied again.

As a first example we discuss an amplifier circuit, already published previously [10]). A pentode amplifier valve P, which shows practically no internal feedback by way of anode-inputgrid capacity, is used for the amplification of a h.f. signal U_t (fig. 5). The feedback is introduced by inserting a selfinduction L_k in the cathode lead, this insertion causes an additional input resistance between the connections 1 and 2 of approximately:

$$\frac{1}{R_f} = \omega^2 L_k C_{kg} S \qquad (16)$$

where C_{kg} is the capacity between cathode and input-grid and S the transconductance of the valve. This input resistance is parallel to R and R_t of fig. 5 and the resulting resistance may be accomodated to

the desired bandwidth B according to eq. (11) by putting:

$$\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_t} = 2\pi BC,$$

where C is the total capacity between the connections 1 and 2. By increasing R as far as possible, a very considerable increase of signal

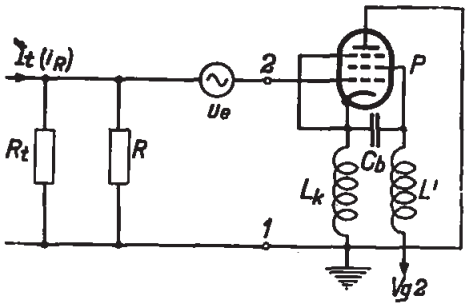

Fig. 5. Circuit used for the application of feedback to a receiver. I_t, i_R, R_t, R and u_e have the same meaning as in fig. 2, 3 and 4, R being the impedance of a tuned circuit. The circuit capacity C is not shown here. P is a pentode, the cathode lead is containing a rather small selfinduction L_k, whereas the screen grid voltage V_{g2} is supplied in series with a choke coil, the screen grid being connected to the cathode by a block condenser C_b.

to noise ratio may be obtained. This was checked experimentally by Ir. F. A. de Groot of this laboratory, who obtained an increase of signal to noise ratio about two-fold by increasing R about four-fold, practically equal to what would be expected from the equation (13a).

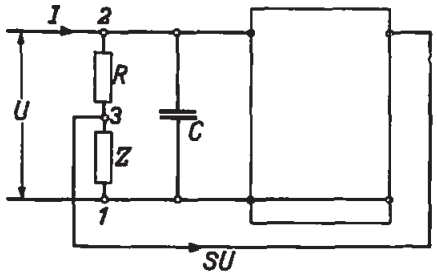

Fig. 6. Circuit used for the application of feedback to an amplifier. I is the signal current from a source with high internal resistance. R is the resistance and C the capacity of the grid circuit, Z being an impedance (in series with R) providing the feedback.

A second example makes use of the circuit shown in fig. 6 especially suited for broad band amplification (e.g. behind a photoelectric cell) [9]). The output current of a four-terminal amplifier is made to

pass through a small impedance Z. Across the input terminals we have an input capacity C. The resistance of the input circuit, represented by R, ($R \gg |Z|$) is taken much higher than corresponding to the desired bandwidth B. The impedance β of section II is then approximately given by:

$$\beta = \frac{-Z}{1 + j\omega CR}. \tag{17}$$

Z_{io} is given by (compare section II):

$$Z_{io} = \frac{R}{(1 + j\omega CR)}, \tag{18}$$

whereas, according to (2):

$$Z_i = \frac{R}{(1 + SZ + j\omega CR)} = \frac{R/(1 + SZ)}{1 + j\omega CR/(1 + SZ)} \tag{19}$$

Taking $Z = j\omega L$ (selfinduction) and S negative, such that $SZ = -j\omega CR$, would yield a real value for the input impedance $Z_i = R$ for the whole frequency range. The condition of negative S is e.g. satisfied by connecting the output of the *second* amplifying stage to the point 3 of fig. 6. Taking $Z = r$ (resistance) and S positive, the input resistance between the points 1 and 2 of fig. 6 may be so much decreased, that the desired shape of the frequency curve is restored. The condition of positive S is e.g. satisfied by connecting the output of the *first* amplifying stage to the point 3 of fig. 6.

Both choices of SZ may yield an equal signal to noise ratio if applied consistently. The restoring of the bandwidth B might also be achieved by other methods, e.g. by amplifying the higher frequencies of the band more than the lower ones in the following stages of the amplifier. According to section II, this method will yield the same signal to noise ratio as our method of applying feedback *).

VI. *Application to devices for measuring minute currents.* Feedback may also be applied to measuring instruments like galvanometers, electrometers etc. [9]).

We first discuss the measurement of minute currents by means of an electrometer-triode [18]). The current I to be measured is sent

*) A contrary statement in a previous publication [9]) was due to the fact, that the capacity C (fig. 6) was neglected in the calculation of the signal to noise ratio with feedback, whereas this capacity was taken into account in the method of amplifying the higher frequencies more than the lower ones.

through the grid circuit of the electrometer triode, consisting of an external resistance R and a parallel capacity C (C being determined by the capacity C_e of the valve and the wiring capacity of the grid circuit), causing a potential drop between grid and ground (fig. 7)

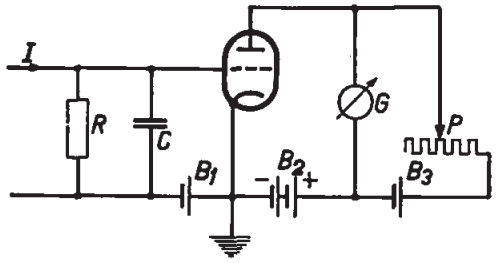

Fig. 7. Schematic circuit with an electrometer-triode, used in measuring extremely small direct currents I. B_1, B_2 and B_3 are batteries, R and C being the resistance and capacity of the grid circuit respectively. G is a galvanometer and P a high variable resistance, having such a value, that the galvanometer deflection without current I is zero.

resulting in a change of the anode current, that may be detected by a galvanometer in the anode circuit of the triode. This potential drop is given by:

$$V = IR\,(1 - e^{-t/CR}), \tag{20}$$

V being zero for $t = 0$, whereas

$$V = V_0 = IR \text{ for } t \gg CR. \tag{20a}$$

We will only discuss here the case, in which the current I is measured by reading the end value of the galvanometer deflection φ, assuming the shot effect of the grid current to be unimportant [18]). The time of measurement is proportional to CR, the larger the value of CR, the longer the time of measurement will be.

The mean square of fluctuating voltage v on the grid without feedback is then given [4]) by:

$$\overline{v^2} = \frac{kT}{C}, \tag{21}$$

as the mean energy stored in the capacity C is $\frac{1}{2}C\overline{v^2} = \frac{1}{2}kT$, according to the law of equipartition [3]). The current I to be measured is causing a drop V_0 in the voltage V_g between grid and cathode, determined by (20a). Therefore the mean square of the error i with which I can be measured is given by:

$$\overline{i^2} = \frac{\overline{v^2}}{R^2} = \frac{kT}{CR^2} = \frac{kT}{R\tau} \tag{22}$$

(τ being equal to CR). Hence the accuracy of measurement is increased by increasing R.

When feedback is applied, the theory of section II does not hold necessarily, because the voltage drop V_o is a direct voltage in contrast with the fluctuating voltage v. We shall show, however, that the problem may be easily solved if we compare:

a) a normal grid-circuit $R = R_o$ and $C = C_o$, $\tau = \tau_o = C_o R_o$.

b) a grid-circuit in which R is raised to a higher value R_1 and afterwards feedback is applied so that the frequency characteristic is exactly the same as in case *a*).

In case *b*) before feedback $R = R_1$ and $C = C_o$, whereas after feedback $R = R_2$ and $C = C_2$, but $\tau = C_2 R_2 = C_o R_o = \tau_o$. Then the voltage drop $V = V_b$ will be changed with respect to the voltage drop $V = V_a$ of case *a*) by a factor R_2/R_o, whereas for the fluctuating voltage under the same notations the relation $\overline{v_b^2}/\overline{v_a^2} = (\overline{i_{R_1}^2}/\overline{i_{R_o}^2}) \cdot (R_2/R_o)^2$ holds. Here i_{R_o} and i_{R_1} are the fluctuating currents in a frequency interval Δ f, which, according to the equivalent circuits of fig. 2, may be supposed flowing into the grid circuit to account for the Brownian motion of electricity in the resistances R_o and R_1 respectively. According to eq. (7*b*) $\overline{i_{R_1}^2}/\overline{i_{R_o}^2} = R_o/R_1$. The improvement in the accuracy of the measurement depends on $V/\sqrt{\overline{v^2}}$.

Comparing *b*) with *a*) yields

$$\frac{V_b}{\sqrt{\overline{v_b^2}}} = \left(\frac{V_a}{\sqrt{\overline{v_a^2}}}\right) \cdot \left(\frac{R_o}{R_1}\right)^{-\frac{1}{2}},$$

so that eq. (22) from

$$\overline{i^2} = \frac{kT}{R_o \tau_o} \quad \text{for case } a) \tag{22a}$$

turns into

$$\overline{i^2} = \frac{kT}{R_1 \tau_o} \quad \text{in case } b). \tag{22b}$$

We now have proved the general theorem:

If the external grid resistance R of an electrometertriode is raised from a value R_o to a higher value R_1 and afterwards the frequency characteristic of the grid circuit is exactly restored to the former one by means of feedback, the mean square of the error in the measured current due to Brownian motion of electricity is decreased by a factor R_o/R_1.

If an electrometertriode or amplifier valve is used with *floating* grid [6] [18]), it is of course practically impossible to raise the grid

resistance R_0 to a higher value R_1. Hence no improvement in the accuracy of measurement can be achieved by this method [6]).

As a possible feedback circuit to obtain the preceeding results the circuit of fig. 6 may be recommended. As the voltage amplification of the triode may be insufficient, it is advised to use one or more stages of amplification behind the electrometer-triode.

We now pay attention to the measurements of small direct currents with the help of a galvanometer. As is wellknown the Brownian movement of a galvanometer may to a great extent be due to the spontaneous motion of electricity in the damping resistance R_d of the galvanometer [16]) [17]). This was experimentally verified by Ornstein and his coworkers [8]), who were able to show that cooling of the damping resistance R_d resulted in a decrease of the Brownian movement of the galvanometer. The influence of cooling is to diminish the spontaneous e.m.f. in series with R_d. Any other resistance, having a mean square of spontaneous voltage across its terminals lower than that of an ohmic resistance at room temperature, would have the same effect. As galvanometers are generally used under the conditions of critical damping, R_d should have a fixed value. The mean square of spontaneous voltage across the terminals of R_d in a small frequency interval Δf is given by

$$\overline{u_R^2} = 4kTR_d \Delta f \text{ according to } (7a).$$

Now those resistances may be provided by a suitable feedback. A very useful circuit may be that shown in fig. 6, where Z is a purely ohmic resistance r. According to section V (compare eq. 19), the input impedance Z_i of the amplifier is given by: $Z_i = R/(1 + Sr)$. We now will match Z_i and R_d by suitably choosing R and Sr. Then

$$R_d = \frac{R}{1 + Sr}.$$

The mean square $\overline{u^2}$ of the spontaneous voltage u across its terminals in a small frequency interval Δf is given by (according to eqs. 3 and 4, compare fig. 4):

$$\overline{u^2} = \overline{u_n^2} + \overline{u_v^2} = \frac{\overline{u_{no}^2}}{(1 + Sr)^2} + \overline{i_o^2}\left(\frac{r}{1 + Sr}\right)^2,$$

or:

$$\overline{u^2} = 4kTR_d \Delta f \left[\frac{1}{1 + Sr} + \frac{R_e}{R_d}\right] \ll 4kTR_d \Delta f, \qquad (24)$$

if $Sr \gg 1$ and $R_e/R_d \ll 1$ (according to eqs. 7*a*, 9, 9*a* and 23).

By eq. (24) it is proved that by means of feedback damping resistances of a galvanometer may be obtained, giving much less Brownian movement of the galvanometer than an ordinary resistance.

If we connect a resistance R' of this kind in parallel to a normal ohmic resistance R, the latter will convey energy to the former. Hence we have obtained a refrigerating device. The effectiveness of this device, however, is very low, as may be seen from the following example.

Let the two resistances R and R' be equal in amount within a frequency range B, whereas $R' = 0$ outside this range. Let the temperatures of R and R' be T and 0° abs. respectively. Then the above transfer of energy amounts to KTB Joules/second, which is always a practically negligible amount. The time required to drop the temperature of R by 1° is e.g. of the order of 10^6 years.

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